Success in Algebra: Puzzle it Out

Building intuition, logic, confidence, and stamina in algebra students

NCCTM State Conference

October 30, 2014

Jane M. Kang

jkang@edc.org

EDC Learning transforms lives.

Who Am I? Puzzles

Who Am I?

$$\frac{\textit{numerator}}{\textit{denominator}} = \frac{\square}{\square}$$

- I am less than ½.
- I am not 0.
- My numerator and my denominator both have one digit.
- My denominator is 3 more than my numerator.
- Neither my numerator nor my denominator is a square number.

Who Am I?

$\square\square\square\square$

- I am less than 1½.
- I'm greater than 1.45.
- Three of my digits are even.
- None of my digits are prime.
- My hundredths digit minus my thousandths digit is 2
- The product of my hundredths digit and my thousandths digit is 48.

Find the coefficients a, b, and c in the quadratic $y = ax^2 + bx + c$ so that the roots are $-\frac{2}{3}$ and $-\frac{1}{2}$ and the minimum value y can attain is $8\frac{1}{8}$.

Transition to Algebra

transition to algebra
make algebra make sense

- A coherent, full-year NSF-funded algebra support curriculum organized around five key ***algebraic habits of mind***

"…what is even more important is to give students the tools they will need in order to use, understand, and even make mathematics that does not yet exist. A curriculum organized around habits of mind tries to close the gap between what the users and makers of mathematics do."

Cuoco, Goldenberg, and Mark, 1996, p. 376

Algebraic Habits of Mind

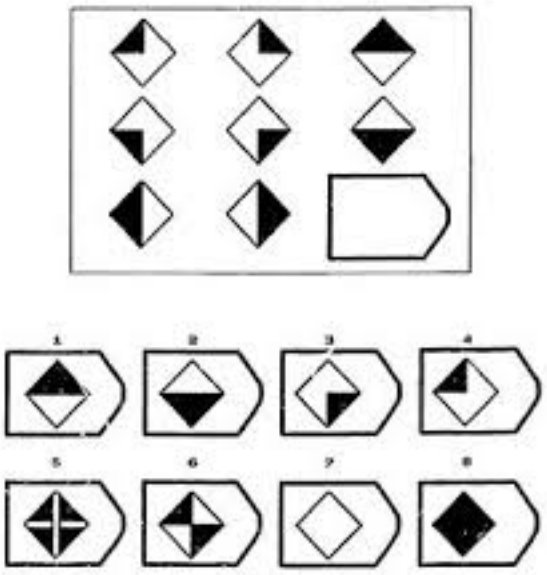
- Puzzling and Persevering
- Seeking and Using Structure
- Using Tools Strategically
- Describing Repeated Reasoning
- Communicating with Precision

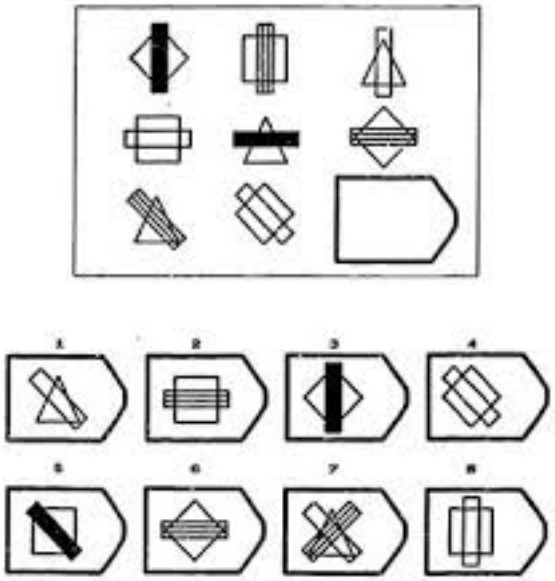
$$\frac{60}{19-x} = 5$$

????

$$19 - x = 12$$

Raven's Progressive Matrices





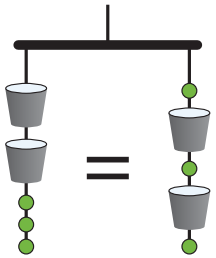
Puzzling and Persevering

- Puzzles as the "main course"
  - Puzzles that *feel* do-able and are challenging
- Developing a puzzler's disposition
  - What-can-I-do? vs. What-am-I-supposed-to-do?
- Two dimensions: Cognitive challenge & Required arithmetic/mathematical knowledge
- Creating puzzles
  - *Producers*, not just consumers, of mathematics

Mobile Puzzles

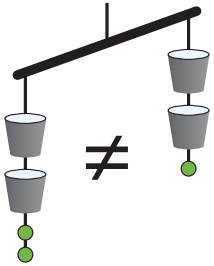
In each of these problems a dot (•) = 1.

This mobile *always balances*. Why?



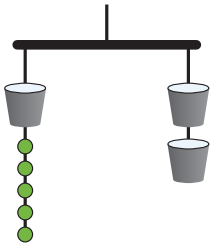
Equivalence

This mobile *never balances* no matter what number the bucket represents. Why?



Inequality

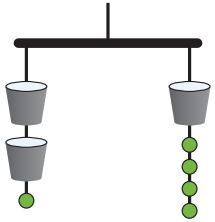
This mobile *only balances when* the buckets represent a certain number.



What number makes it balance?

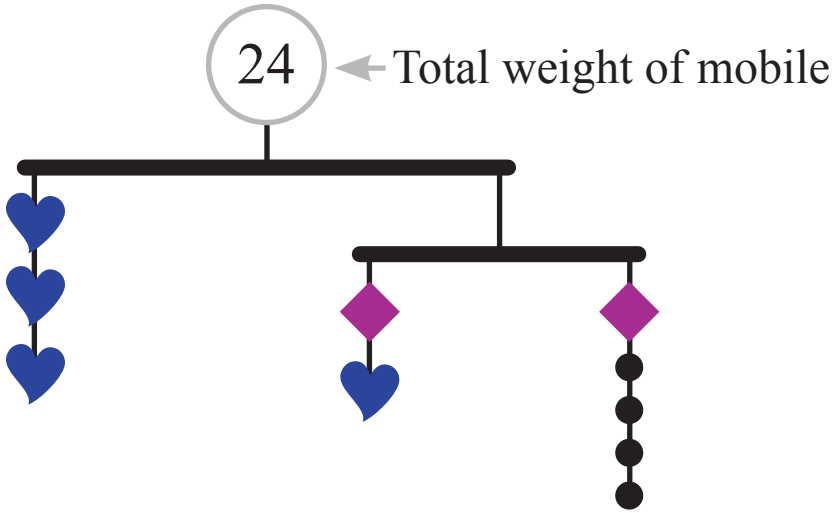
Solving

Does this mobile balance *always, sometimes, or never?* If sometimes, *when?*



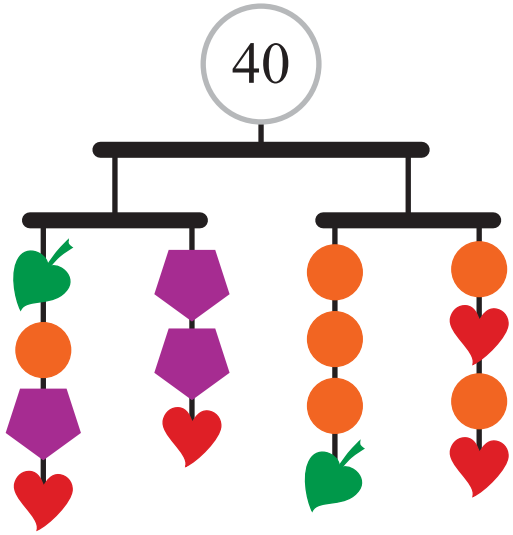
Logical Solving

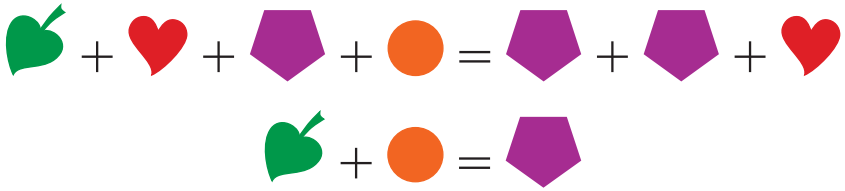
Mobile Puzzles

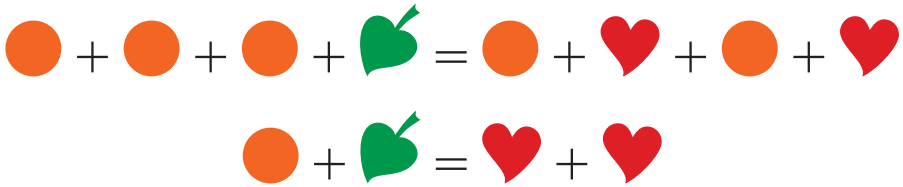


♥ = 4 ◆ = 2 ● = 1

Mobile Puzzles







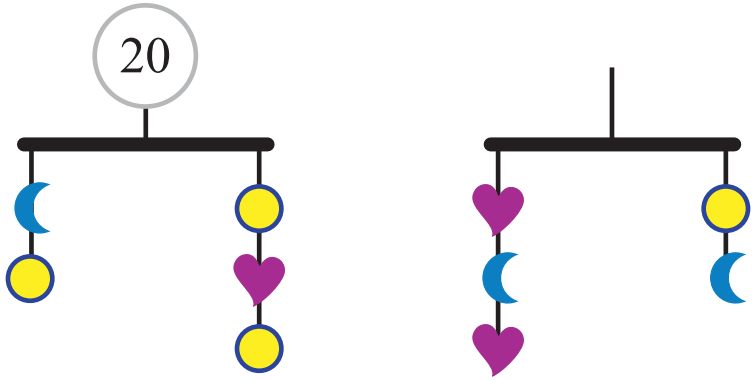
 = _____

 = _____

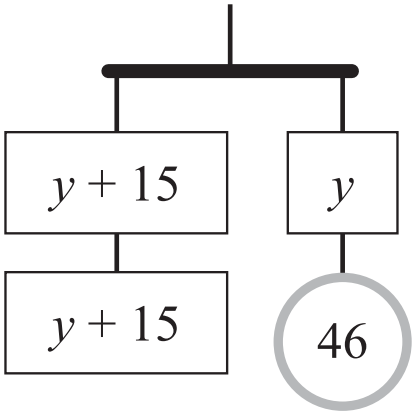
 = _____

 = _____

Mobile Puzzles

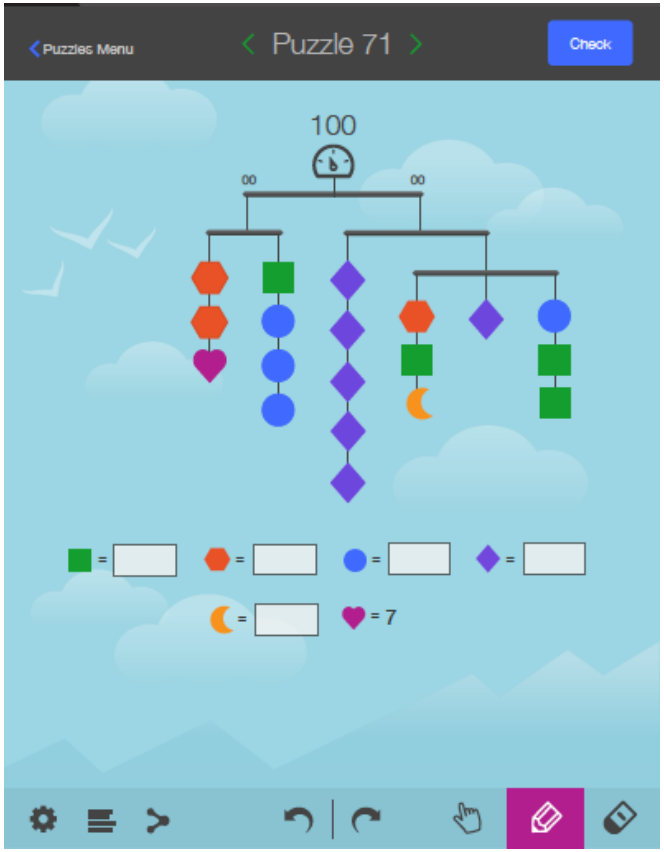


♥ = ___ ● = ___ ☾ = ___



$y =$ ____

Mobile Puzzle App: SolveMe

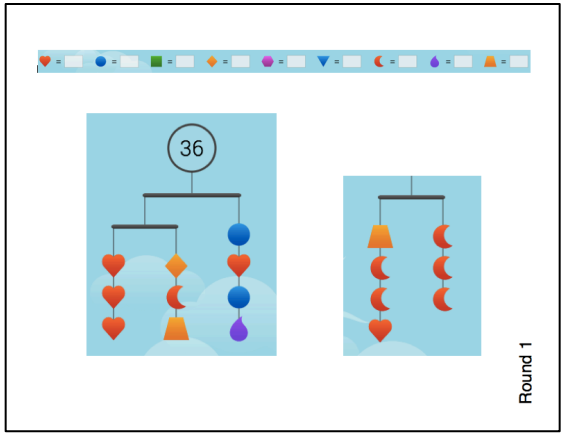


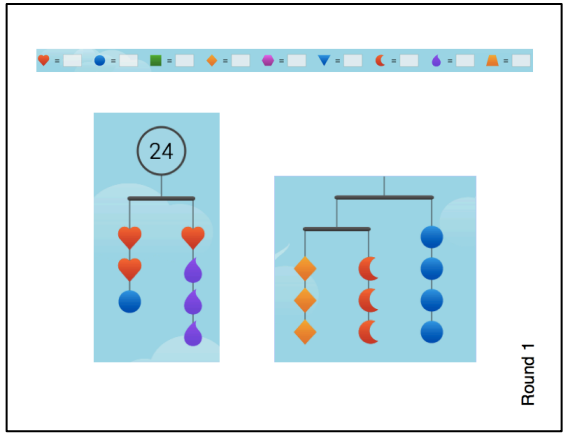
Interactive puzzling features:

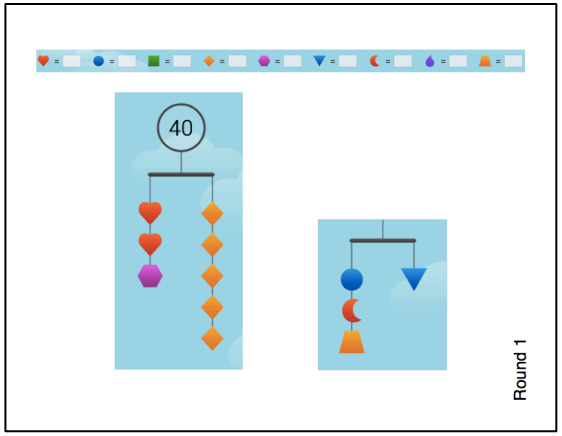
- Shape equations
- Subtraction, division, factoring
- Substitution
- Annotations
- Sharing
- “Build Your Own” mode

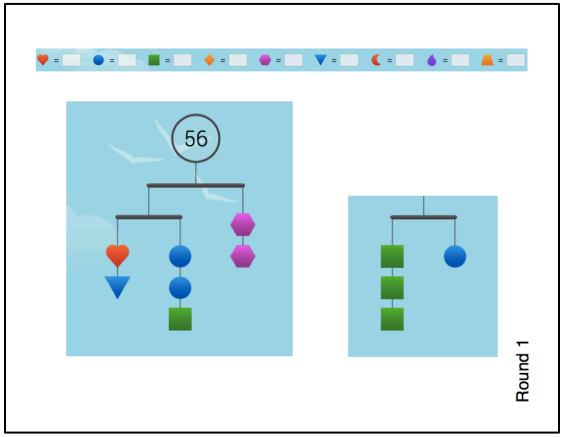
solveme.edc.org

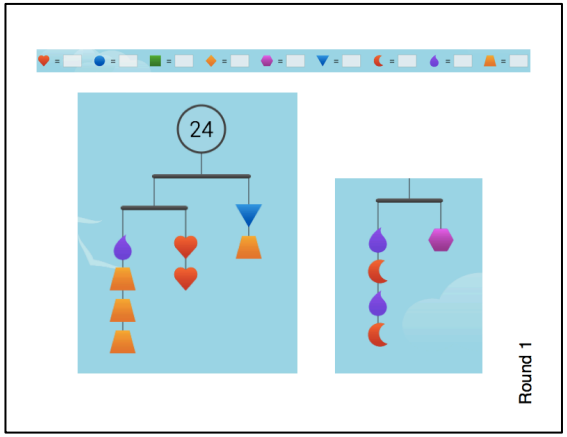
Mobiles: Collaborative Game











Mystery Number Puzzles

③ What could ☆, 💧, and ⬢ be if all the variables represent different numbers?

💧 • ⬢ = ☆

⬢ + ⬢ = ☆

💧 + 💧 + 💧 = ☆

☆ = _____

💧 = _____

⬢ = _____

Mystery Number Puzzles

⬟ + ⬟ = ⬟	Only one solution
⬡ • ⬡ = ⬡	Two solutions
▲ • ■ = ▲	For some value of ▲, ■ can have any value. For some value of ■, ▲ can have any value.
♦ + ♦ = ● ♦ • ♦ = ●	Two solutions (assuming different variables can have the same value)

Latin Squares Puzzles

- Use the clues to fill in the grid so that every row and every column contains one of each element.

a*, *b*, *c Latin Square

c		a
	c	

MysteryGrid Puzzles

- In MysteryGrid puzzles, the numbers in each "cage" should reach the target number using the given operation.
- For example, a 3-cell, "20, ×" cage means you need to fill that cage with 3 numbers that multiply to 20.

MysteryGrid **1, 3, 4, 5**

4, +		4, ÷	1, –
20, x	12, +		
			2, –
	15, x		

MysteryGrid Puzzles

MysteryGrid **2, *x*, 2*x***

$2x+2$, $+$	$2x^2$, $\bullet$	
		$4x$, $\bullet$
$3x$, $+$		

MysteryGrid ***x*, 2*x*, 4*x***

$8x^2$, $\bullet$	$3x$, $-$	
	$5x$, $+$	
$4x^2$, $\bullet$		

MysteryGrid Puzzles

MysteryGrid a, a^2, a^3, a^4

a^6, •			$2a^4+a^3$,+
a^7, •	a^4, •		
		a^5, •	
	a^7, •		

Transition to Algebra

transition to algebra

make algebra make sense

EDC Learning transforms lives.

firsthand HEINEMANN DEDICATED TO TEACHERS

Full-year algebra-support curriculum with student & teacher materials that supports the Common Core Standards for Mathematical Practice

For for information: transitiontoalgebra.com or jkang@edc.org